Student's Name

Professor's Name

Courses

Date

Questions

1. Positional Logic and Agent Modal Logic (25 points)

(a) Use a truth table to show whether $\models (p \vee q)\ ((p \wedge r) \vee (q \wedge \neg r))$. Your answer follows from the truth table.

(b) Translate the following sentences from English to modal logic. (5 points)

• "If it is possible that 2+2=4 is necessarily true, then it is necessary that 2+2=4 is possibly true."

• "If I know that I know that 2 is a prime number, then I know that I don't know that 2 is not a prime number."

• "It is either legally permissible to cross the street, or legally permissible not to cross the street."

• "Forever in the future, if it is Monday, then there will be a time in the future when it is not Monday."

(c) Translate the following formula of modal logic into English, using the specified meaning of the box/diamond. (5 points)

Alethic: $\blacklozenge(p \rightarrow q) \vee \neg q$

Deontic: $\blacklozenge(p \vee r) \wedge \neg p$

Epistemic: $p \vee (q \rightarrow \neg\neg p)$

Temporal: ♦(p ∧ (q → r))

(d) Translate the following subsumptions into description logic. (5 points)

Every parent is a person who has at least one child that is also a person.

The distal phalange is the bone that is located in the fingertip.

Every dog is either not a pet or a good boy.

A partial order is a relation that is reflexive, antisymmetric and transitive.

(e) e) Translate the following concepts from description logic to English. (5 points)

dog t ¬∃hasPet.(dog u large)

¬person u ∀hasChild.∃hasChild.⊥

movie u ∀hasGenre.horror u ∃hasDirector.>

movie u ∃hasGenre.horror u ∀hasDirector.⊥

Solutions

(a)

P	q	r	P	v	q	r	P	v	(r -q)
0	0	0	0	0	0	0	0	1	1 1 0
0	0	1	0	0	1	1	1	0	1 0 1
0	1	0	1	0	1	1	0	1	0 0 1
1	0	1	1	0	1	1	1	0	1 0 1
1	1	1	1	1	1	0	0	1	0 1 1

In every row, the formula (p ∨q) ((p ^ r) ∨ (q ^ ¬r)) is assigned a truth value of 1. Therefore, this formula is true regardless of the truth values of its atoms, so ⊨ (p ∨q) ((p ^ r) ∨ (q ^ ¬r)).

(b)

Q(Q*p* ⋁♦Q*q*)

(Q*p*⋁♦Q*q*)”

(Q*p*⋁♦Q*q*)”

(QPv♦q)

(Q ∨P)

(C) is it necessarily true that

I know that, I consider it possible that

At every point in future, will be true later

It is mandatory

It is not permitted for forbid

(d)

Q*p* →♦*p*

Q♦*p.*

Qp →*QQp.*

¬♦*Q*¬*p.*

*(*e) ”the is a pet , the is large“

The person has a child, the child is for the person

The movie is of the horror genre the director is for the movie

The movie is of the horror genre, it has a director

2. Single Agent Modal Logic (25 points)

(a) Determine whether (♦q ∧ ♦p) → ♦♦q is valid. If it is valid, explain why it is valid. If it is not valid, provide a counterexample and show that it is a counterexample

(b) (b) Let M = (W, R,V) be given by W = {w1, w2, w3}, R = {(w1, w2), (w2, w3), (w3, w1), (w3, w3)}, V(p) = w3. Draw this model

(c) For the model described in (c), explain whether M, w3 |= ♦♦(p ∨ q)

(d) Consider the formula p ∨ ♦¬p. Is it a substitution instance of a validity of propositional logic? If it is, provide the valid formula that it is a substitution instance of. If it is not, provide a (non-valid) formula of propositional logic that it is a substitution instance of

(e) Consider the formula ¬p ∨ ♦p. Is it a substitution instance of a validity of propositional logic? If it is, provide the valid formula that it is a substitution instance of. If it is not, provide a (non-valid) formula of propositional logic that it is a substitution instance of.

(f) Is the following a correct proof in the system K? If not, list all line numbers where there are errors. (5 marks) 1. ♦p ∨ ¬p T 2. q → (♦p ∨ ¬p) T(1) 3. (q → (♦p ∨ ¬p)) → (q → (♦p ∨ ¬p)) K 4. (q → (♦p ∨ ¬p)) MP(2,3) 5. (q → (♦p ∨ ¬p)) Necc(4)

Solutions

(a) The formula is valid. Take any pointed model (♦q ∧ ♦p) → ♦♦q. Suppose that the first disjunct is not satisfied in :(_a_bp ^ _a_bp). Then _a_bp ^ _a_bp.

So we have j= _a_bp and j= _a_bp. Because j= _a_bp, we know

that w has at least one a-successor w1. Then, j= _a_bp, we have j=

_bp. This implies that w1 has at least one b-successor w2 such that M;w2 j= p.

(b) W = fw1;w2;w3;w4g,

Ra = f(w1;w1); (w1;w4); (w4;w1); (w4;w4); (w2;w2)g,

Rb = f(w1;w3); (w3;w1); (w4;w2); (w2;w2)g,

V (p) = fw2;w4g and

V (q) = fw1;w3g

(c) We have w2 2 V (p), so M;w2 j= p. Since w2 is the only a-successor of w2, it follows that M;w2 j= _ap. Because w2 is a b-successor of w4, we then have M;w4 j= _b_ap.

(d) We have w3 62 V (p), so M;w3 6j= p. Since w3 is the only b-successor of w1, it follows that M;w1 6j= _bp. Because w1 is an a-successor of itself, we then have M;w1 6j= _a_bp. Furthermore, w1 62 V (p), so M;w1 6j= p. Together with our previous conclusion M;w1 6j= _a_bp, this implies that M;w1 6j= p _ _a_bp. Since w1 is an a-successor of w1, it follows that M;w1 6j= _a(p _ _a_bp).

(e) ♦ ap ^♦aq

(f)

$n-\ 4\ Q(\neg(p \vee q) \rightarrow \neg p)$

$n-\ 3(n-4) \rightarrow (n-2)$

$n-\ 2\ Q\neg(p \vee q) \rightarrow Q\neg p$

$n-\ 1(n-2) \rightarrow (n)$

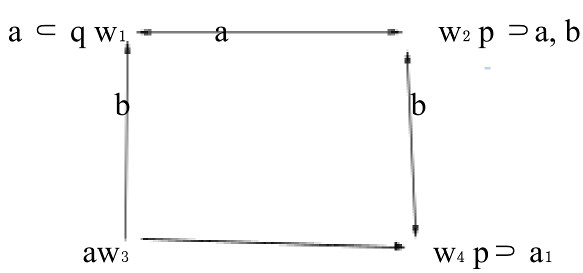
$n\neg Q\neg p \rightarrow \neg Q\neg(p \vee q)$ k

MP$(n-3, n-4)$

T MP$(n-2, n-1)$

3. Multi Agent Modal Logic (25 points)

A



(a) Give the formal description of this model M, i.e. M = (W, R,V), where W = · · · , Ra = · · · , Rb = · · · and V = · · · . (3 marks)

(b) Explain whether M, w4 |= ap $\wedge$ bp. (5 marks)

(c) Explain whether M, w3 |= a(p → b♦bp). (5 marks)

(d) Give a formula that holds on w1 and w4 but not on w2 and w3. (You do not need to explain your formula.) (4 marks)

(e) Determine whether the formula a(b¬p ∧ b¬q) ∨ ♦a♦b(p ∨ q) is valid. If it is valid, explain why it is valid. If it is not valid, provide a counterexample and show that it is a counterexample.

Solutions

(a) W = fw1;w2;w3;w4g,

Ra = f(w1;w1); (w1;w4); (w4;w1); (w4;w4); (w2;w2)g,

Rb = f(w1;w3); (w3;w1); (w4;w2); (w2;w2)g,

V (p) = fw2;w4g and

V (q) = fw1;w3g

(b) We have w2 2 V (p), so M;w2 j= p. Since w2 is the only a-successor of w2, it follows that M;w2 j= _ap. Because w2 is a b-successor of w4, we then have M;w4 j= _b_ap.

(c) We have w3 62 V (p), so M;w3 6j= p. Since w3 is the only b-successor of w1, it follows that M;w1 6j= _bp. Because w1 is an a-successor of itself, we then have M;w1 6j= _a_bp. Furthermore, w1 62 V (p), so M;w1 6j= p. Together with our previous conclusion M;w1 6j= _a_bp, this implies that M;w1 6j= p _ _a_bp.

Since w1 is an a-successor of w1, it follows that M;w1 6j= _a(p _ _a_bp).

(d) _ap ^ _aq.

(e) The formula is valid. Take any pointed model M;w. Suppose that the first disjunct is not satisfied in M;w, so M;w 6j= :(_a_bp ^ _a_bp). Then M;w j= _a_bp ^ _a_bp.

So we have M;w j= _a_bp and M;w j= _a_bp. Because of M;w j= _a_bp, we know that w has at least one a-successor w1. Then, by M;w j= _a_bp, we have M;w1 j= _bp. This implies that w1 has at least one b-successor w2 such that M;w2 j= p.

4 (Description Logic)

Consider the ABox A given by

• alice : A u ∀r.B

• alice : ¬∀r.F

• bob : ¬C

• (alice, bob) : r and the TBox T given by

• A ≡ ∃r.D •

B v E

• C ≡ ¬D u E

• F ≡ ¬¬A and let K = (A, T).

Using a tableaux-based 6-step method, show whether K is consistent?

(a) Eliminate ⊆ from the TBox (4 points)

(b) Expand the box (4 points)

(c) Estimate defined concepts from the ABox (2 points)

(d) Put the ABox in a navigation form (2 points)

(e) Apply the completion rules of the ABox (8 points)

(f) Check the leaves of the tree that you made in (e) for contradictions, and explain whether K is consistent (5 points)

Solutions

(a) Let A0 = A [flalice : :bob, and K0 = (A0; T). Then K j= Alice : bob if and only if K0 is not consistent.

(b) The new TBox is given by

bob _ :alice u 9bob:alice

Bob l _ :alice u :9alice r:> u bob_

bob _ alice (bob t alice)

(c) The new TBox is given by

Bob _ :alice u 9r:bob.alice

Love 1 _ :bob u :9Alice:> u lovers

alice _ :alice u :9bob:> u

(d) Eliminating defined concepts from A0 yields

Alice: ((:bob and Alice)t

(: bob u :2lover:> u Alice_ u (lovers))) u :love)

_(;Alice) : bob

bob : : Alice

(e) The new ABox is given by

Alice: ((:lovers 9bob.Alice)t

(bob u 8 Alice:? U lovers_ u (bob))) u :Alice u :bob

5 (Epistemic Logic)

In epistemic logic, we only consider those models that are reflexive, transitive and euclidean.

(a) Draw a model that is reflexive and transitive, but not Euclidean. (3 points)

(b) Draw a model that is Euclidean, but neither transitive nor reflexive. (3 points) Now, consider the following situation: a king has given three of his advisers a puzzle, in order to test their problem solving skills. The advisers a, b and c are placed in a line, one behind the other. Adviser c is facing the wall, and can see neither herself nor either of the other advisers. Adviser b can see adviser c, but not himself or adviser a. Adviser a, finally, can see b and c but not herself. This setup is common knowledge among the agents. The king then explains the puzzle to his advisers.

"I have here three red hats and one green hat. I will put three of these hats on your heads. You are not allowed to look at your own head or turn around, and I will not show you the colour of the fourth, unused, hat." He then puts the hats on his advisers.

(c) Draw a model M that represents the situation described above. Use atoms ra, ga to represent a having a red or green hat, respectively, and use rb, gb, rc, gc similarly to denote the colour of the other agents' hats. (5 points)

(d) Explain, using the model that you have drawn, whether c knows the colour of her hat. (3 points) The king then asks the adviser "do you know the colour of your hat?" To this, the answer is "no". (e) The answer by a can be seen as a public announcement. What is the formula ϕ that is being announced? (3 points) (f) The announcement [ϕ] changes the situation. Draw the model M * ϕ that represents the new situation. (4 points) The king then asks a "does adviser c now know the colour of her hat?" (g) What does an answer? Explain your answer using the model M * ϕ that you just drew. (3 points) (h) What is the colour of c's hat?

Solutions

In every world where ra is false, so in the even-numbered worlds, there is at least one b-successor where ra is false. So in those worlds, Kbra is also false. Every world has at least one a-successor where ra is false, so in that successor, Kbra is also false. So in every world, KaKbra is false

Positive introspection (i.e., if you know something then you know that you know it).